Geometric substructures modelled on pairs of Hermitian symmetric spaces of the compact type

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Abstract. Let Z and X be uniruled projective manifolds of Picard number 1 such that the respective variety of minimal rational tangents (VMRT) at a general point satisfies a nondegeneracy condition on the second fundamental form. In 2001 Hwang and Mok established the equidimensional Cartan-Fubini extension principle, according to which a germ of VMRT-preserving holomorphic map $f : (Z, z_0) \to (X, x_0)$ must necessarily extend to a biholomorphism $F : Z \to X$. In 2010, Hong and Mok extended this to the nonequidimensional case for germs of holomorphic immersions between uniruled projective manifolds, allowing $\dim(Z) < \dim(X)$, by proving that f must necessarily extend to a rational map provided that a certain relative version of the nondegeneracy condition on the second fundamental form is satisfied. Recently, Mok and Zhang (2017) developed the theory of geometric substructures by considering germs of complex submanifolds of $(S, x_0) \hookrightarrow (X, x_0)$ and introducing geometric substructures on S by taking intersections of the VMRTs of X with projectivized tangent spaces of S. We introduced a new relative nondegeneracy condition related to the second fundamental form and proved the extendibility of the germ (S, x_0) to a projective subvariety $Y \subset X$ under the assumption that X is uniruled by lines, i.e., by rational curves whose homology classes are the positive generator of $H^2(X, \mathbb{Z}) \cong \mathbb{Z}$. We achieved this by recovering Y as the image under a tautological map of a certain universal family of chains of minimal rational curves. When the ambient manifold is an irreducible Hermitian symmetric space of the compact type and the germ of geometric structure is modelled on a Hermitian symmetric submanifold, for pairs defined by marked Dynkin subdiagrams (e.g. sub-Grassmannians of rank ≥ 2 in Grassmannians) we recover by differential-geometric means rigidity results (Schubert rigidity) of Bryant, Walters and Hong obtained by cohomological methods. Very recently, our method has been extended to admissible pairs nondegenerate for substructures which are not of the sub-diagram type (e.g., Lagrangian Grassmannians in Grassmannians of the same rank ≥ 3).