

**Holomorphic isometries with respect to the Bergman  
metric and uniformization problems on  
bounded symmetric domains**

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**Abstract.** The study of holomorphic isometries between Kähler manifolds with real-analytic potential functions dated back to Bochner and Calabi. Especially, in his seminal work on holomorphic isometries in 1953 in which the *diastasis* was introduced, Calabi established results on existence, uniqueness and analytic continuation of holomorphic isometries into (finite and infinite dimensional) Fubini spaces. Specializing to the case of bounded domains equipped with the Bergman metric, one derives immediately results of analytic continuation of germs of holomorphic isometries  $f$  with respect to the Bergman metric, and the questions remained as to whether analytic continuation persists across the boundary. In 2012, the author solved the problem of boundary extension in a very general context, proving in particular that  $\text{Graph}(f)$  extends to an affine algebraic variety provided that Bergman kernels are rational functions, which applies in particular to the case of germs of holomorphic isometries from the complex unit ball  $\mathbb{B}^n$  into bounded symmetric domains  $\Omega$  in their standard embeddings. Together with S.-T. Chan the author studied asymptotic behavior of holomorphic isometries of the Poincaré disk into  $\Omega$  and proved their asymptotic geodesy at general boundary points. Boundary curvature estimates lead to various results arising from the study of uniformization problems concerning subvarieties on bounded symmetric domains. This includes answers to the question of characterization of totally geodesic algebraic subvarieties on quasi-projective varieties  $X_\Gamma$  uniformized by bounded symmetric domains in terms of bi-algebraicity, a result which was established when  $\Gamma \subset \text{Aut}(\Omega)$  is an arithmetic lattice by Ullmo-Yafaev (2011) by means of the celebrated result of André-Deligne on monodromy groups of quasi-projective subvarieties of  $X_\Gamma$ , and results on the Ax-Lindemann conjecture for not necessarily arithmetic lattices concerning the characterization of Zariski closures of images of algebraic subsets under the uniformization map. Again the Ax-Lindemann conjecture in the arithmetic case was settled by Klingler-Ullmo-Yafaev (2016) after works of Ullmo-Yafaev and Pila-Tsimerman. We will explore the direction of research which brings together methods of complex differential geometry in the study of holomorphic isometries between Kähler manifolds and methods involving moreover complex analysis and algebraic geometry in the study of uniformization problems concerning algebraic subsets and more general classes of complex-analytic subvarieties of bounded symmetric domains.